

Origami Demonstrations of Area Formulae for General Triangles, Parallelograms, and Trapezoids

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Abstract

Origami demonstrations of area formulae for triangles, parallelograms, and trapezoids involve folding the starting polygon shapes into double-layer rectangles whose heights (altitudes) and areas are one-half those of the polygon. Such demonstrations are typically presented in school textbooks for acute base angles of triangles and parallel sides of parallelograms and trapezoids that are not fully to the right or left of one another. In these special cases, two or four folds for triangles, and three folds for parallelograms and trapezoids, suffice for forming the double-layer rectangles.

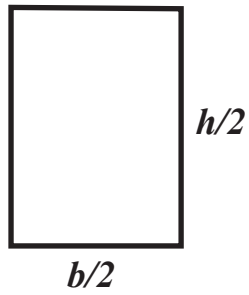
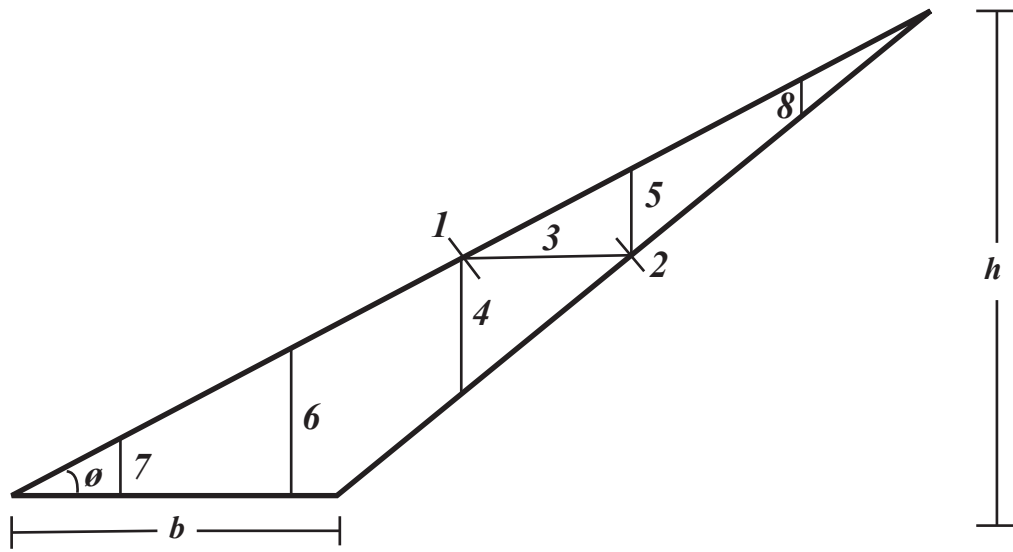
In this paper, we outline origami demonstrations of area formulae for general triangle, parallelogram, and trapezoid shapes, with *any* side of the triangle or parallelogram taken as the base. We also give the minimum number of landmarked folds required to form the double-layered rectangles as functions of the polygon shape parameters. The determination of the number of folds may be useful as the basis for various student explorations of paper-folding aspects of planar geometry and trigonometry.

In the case of a triangle of base length b , altitude h , and measure θ of one of the internal angles at the base, we find the following results for the minimum number of landmarked folds to form the double-layer rectangle:

$0 < h < b \tan \theta$	4 folds,
$h = b \tan \theta$	2 folds,
$b \tan \theta < h \leq 2b \tan \theta$	5 folds,
$nb \tan \theta < h \leq (n+1) \tan \theta, n \geq 2$	$2(n+2)$ folds,

where n is a positive integer.

Similar results are obtained for parallelogram and trapezoid areas.



Sequential landmarked crease lines 1-8 for forming double-layer rectangle of area $bh/4$ in the case of a triangle of base b , altitude h , and internal left base-angle measure θ , with $2b \tan \theta < h \leq 3b \tan \theta$. The initial pinch creases 1 and 2 divide in half the left and right sides of the triangle, respectively.